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EVALUATION OF THE PREDICTIVE CAPABILITY OF A TIME-VARYING MARKOV SWITCHING AUTOREGRESSIVE MODEL

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Abstract: The usefulness of a Markov switching autoregressive model with time-varying regimes in making economic predictions is examined in this paper. The predictive capability of this model was confirmed using data on the Nigerian All-share index and three other macroeconomic variables such as crude oil price, exchange rate and inflation rate from 2006 to 2022. Economic cycles and structural breaks can be detected using Markov switching algorithms, which are becoming popular in econometrics. The study intends to develop a time-varying Markov Switching model with autoregressive components to evaluate the possibility of a market turning from bullish to bearish and vice versa using crude oil prices, inflation rate, and currency rate. By visualizing smoothed regime probabilities, the model is also used to identify turning points or cycles in the All-Share index The analysis shows that the exchange rate, which has a negative impact, is connected with statistical significance in the chosen macroeconomic variables alongside crude oil prices, which have both negative and positive effects, demonstrating a significant connection with the All-share index. The complicated interdependencies between bullish and bearish states are revealed by transition probability matrices, with changing probabilities. Additionally, there are differences in the probability of changing between these states, and smoothed regime probabilities reveal significant changes in market mood, particularly in January 2008, late 2008, early 2009, April 2013, March 2015, and early 2021.

Keywords: All-Share Index, Crude Oil Price, Exchange Rates, Markov-switching Algorithms, Inflation Rate

JEL Classification: C510, E400, E520

1. INTRODUCTION

Over time, the stock market, particularly the equity market, has gained popularity due to its potential for generating investment returns. It holds a crucial position in the economy, and any alterations in this market can have a substantial impact on the financial well-being of individuals, corporations, and even the overall economic health of nations (Agwuegbo, Adewole, and Maduegbuna, 2010). In economies, the stock market serves multifaceted purposes that contribute significantly to economic growth and advancement. It acts as a platform for the allocation and distribution of capital. In modern economies, the stock market plays a central role, serving various purposes that contribute to economic expansion and progress. It serves as an efficient means for forming and distributing capital. Through the stock market, both governments and businesses can raise long-term capital to finance new projects, expand their operations, and modernize their commercial and industrial activities. A country's stock market provides essential infrastructure, including trading platforms and a conducive environment, for a diverse group of participants. These participants include individual investors and institutional entities engaged in trading various financial assets, such as stocks and bonds. Essentially, a stock exchange functions as a structured institution where securities issued by listed companies can be openly bought and sold. One of its primary functions is to act as an intermediary, facilitating the flow of funds between individuals who save and entities in need of capital. The significance of the stock market within a nation plays a pivotal role in fostering economic progress and advancement. It helps mobilize domestic resources within the economy and channels them toward productive investments. The performance of a stock market is typically evaluated through its market index, which can be influenced by a wide range of factors, including economic conditions, political stability, societal and cultural factors, and international developments (Erdogan and Ozlale, 2013).

The Nigerian Stock Exchange market (NSE) encompasses 19 sectors, comprising a total of 47 industries and 185 individual stocks. These sectors and industries collectively constitute the Nigerian equity market, offering a diverse array of investment opportunities. The NSE is a significant and fully integrated market infrastructure in Africa, serving the continent's largest economy. It plays a crucial role in facilitating companies' access to capital for funding their business expansion. Additionally, the NSE provides a wide range of regulated securities to cater to the investment objectives of its domestic, regional, and international investor base. The NSE market consists of 19 sectors, comprising a total of 47 industries and 185 individual stocks. These sectors and industries collectively make up the Nigerian stock market, providing a diverse range of investment opportunities. A significant milestone occurred in March 2021 when the NSE underwent a transformation, transitioning from a nonprofit organization to becoming a corporation known as the Nigerian Exchange Group Plc. This structural change reflects the dynamic evolution of the financial market. The history of the NSE dates back to 1961 when it commenced operations with only 19 securities trading on its floors, and since then, it has experienced remarkable growth. As of the most recent data available, there are 139 listed equities on the Exchange. The companies listed on the NSE represent a diverse cross-section of the economy, spanning sectors from agriculture and manufacturing to services. Many of these listed companies have affiliations with foreign or multinational entities, reflecting the globalized nature of the Nigerian economy and its integration into the international business landscape. The All-share index (ASI) is a crucial metric in the financial world, especially for stock markets. It serves as a comprehensive indicator of the market's overall performance, taking into account several key factors such as market capitalization, liquidity, turnover ratio, and the price-to-earnings ratio (P/E ratio). Analysts and investors often rely on the ASI to forecast market growth and assess the health of a stock exchange. The NSE's ASI is a broad-based index that reflects the collective trading behavior of common shares listed on the exchange. This index is calculated daily and provides insights into the direction of price movements in the market. It had an initial value of 100 in January 1984. Market indices, including the ASI, offer a straightforward means of assessing the overall changes in a market's direction and magnitude. Even in bullish or bearish markets, individual stocks may not always move in tandem with the general market trend. Different stocks will experience price fluctuations on a typical trading day, some gaining and others losing. Predicting the overall market trend based solely on individual stock movements would be a cumbersome task. This is where a readily available market price index proves invaluable. A market index is a statistical measure that reflects the total value of a market attribute, in this case, the market's price performance. The index value condenses this information into a single statistic, making it easier to gauge the overall market direction. Various methods can be used to calculate indices, including price-weighted, capitalization-weighted, and share-weighted indexes. The choice of the index's basis depends on the desired outcome and the specific characteristics of the market being analyzed. Understanding the basis of an index is crucial when using it for analysis or investment decisions. Whether it represents the entire market, a specific sector, or a subset of stocks, knowing how an index is constructed and weighted provides valuable insights for making informed financial choices. The ASI in the equity market is influenced by various macroeconomic factors that extend beyond the boundaries of the capital market. Capital markets are often considered the pulse of any economy due to their ability to rapidly respond to fundamental economic shifts. Daily news reports can provide insights into the stock market's performance as an indicator of economic health. News coverage frequently includes updates on markets worldwide, revealing how competitive different economies and equities are. The size and movement of stock prices, market indices, and liquidity often reflect changes in macroeconomic conditions. Equity market prices are shaped by investors' expectations and judgments based on available information, making stock market forecasting a field marked by its challenges and uncertainties. Accurate prediction of stock market values or trends is crucial for making informed investment decisions in a dynamic global economy. Equity investors are drawn to the stock market because they can anticipate returns based on a range of financial and macroeconomic factors. Recently, more attention has been given to the ASI as a means of evaluating a segment of the Stock Market. The general investing public considers the ASI a significant indicator and uses it as a benchmark to assess the returns of their portfolios or those managed by fund managers. Equity market investments are typically long-term in nature, and any change that could significantly impact the overall stability of the economy usually has a substantial effect on its performance. For instance, inflation is recognized as a factor that can influence market performance, as acknowledged by Corrado and Jordan (2002). Inflation has long been recognized as one of the primary factors that can seriously harm an economy. The monetary authorities in Nigeria are continually seeking solutions to address inflation. Therefore, research on how macroeconomic variables such as the inflation rate affect stock market performance can be beneficial for both investors and policymakers, as the economy's performance is closely tied to stock market performance. Interest rates are another significant macroeconomic variable directly linked to economic growth. Interest rates typically represent the cost of capital, whether it is interest payments on borrowing and lending. The interest rate is a crucial factor that affects investment decisions, borrowing costs, and overall economic activity. Exchange rate stability is also vital, as it can curb foreign exchange market speculation and reduce capital flight, as noted by Garba (1997). Additionally, exchange rate fluctuations can impact domestic stock markets differently based on a country's trade dynamics. Research has shown varying opinions on the connection between exchange rate volatility and equity markets, with some studies supporting a link and others arguing against it.

This work focuses on the NSE ASI with the aim of forecasting its movements by fitting a Markov-switching model to the series. Decision-makers and investors can benefit greatly from understanding how macroeconomic conditions and other factors affect the ASI. To determine the possibility of a market turning from bullish to bearish and vice versa, the study specifically uses crude oil prices, inflation rate, and currency rate to build a time-varying Markov switching model with autoregressive components. By visualizing smoothed regime probabilities, the model is also utilized to identify ASI cycles or turning points.

2. RELATED STUDIES

The following are summaries of major findings from selected studies: Aigbovo and Izekhor (2015) examined the effect of currency rates, inflation rates, interest rates, money supply, industrial output index, and international oil prices on the Nigerian equity market index. Their research found substantial links between these macroeconomic factors and the equity market index. Adamu and Gbande (2016) studied the connection between Nigerian inflation rates and equity returns, discovering a positive and statistically significant relationship. The GARCH-MIDAS technique was used by Asgharian, Hou, and Javed (2013) to investigate the influence of several macroeconomic factors on equity market volatility in the United States. They discovered that taking macroeconomic factors into account enhanced the forecast of equity volatility. Neifar et al. (2021) examined the linkage between macroeconomic variables (interest rate, CPI, and so on). Their studies revealed that there were considerable effects on stock market performance. Robert (2008) found no substantial association between BRIC equity market index prices and macroeconomic factors (exchange rate and oil price). Endang and Reminta (2021) evaluated the influence of macroeconomic factors on Brazilian stock prices (inflation, currency rate, interest rate, and money supply), demonstrating a positive association with money supply. Ratanapakorn and Sharma (2007) discovered substantial connections between the US equity market index and macroeconomic variables such as industrial production, inflation, money supply, interest rates, and exchange rates. Shaoping (2008) discovered a substantial positive link between changes in the Chinese money supply and stock prices. Narayan and Narayan (2012) investigated the impact of currency rates. Venkatraja (2014) investigated the impact of macroeconomic variables on the Indian equity market, discovering substantial correlations between industrial production, the wholesale price index, foreign institutional investment, the real effective exchange rate, and gold prices. Aweda, Are, and Akinsanya (2014) explored the link between macroeconomic variables and equity market returns in the United Kingdom and the United States, discovering substantial long-run correlations between stock returns and a variety of parameters. Alabi and Bada (2022) used a two-state Time-varying Markov-Switching model to investigate the linkage between crude oil prices, foreign reserves, and exchange rates, and discovered insights into exchange rate transition stages. These studies contribute to a complete knowledge of the links between macroeconomic factors and equity market performance in various nations.

3. METHODOLOGY

This study adopted a secondary method of data collection where the exchange rate, inflation rate, and crude oil price were retrieved from a database of the Central Bank of Nigeria and the All-share index (ASI) was retrieved from the National Bureau of Statistics (NBS) database. The period covered between 2006-2022, which contains 201 observations. The data is analyze using the time-varying Markov-switching with autoregressive dynamics. Markov switching models are a class of statistical models used to capture regime changes or shifts in the underlying data-generating process. These models are particularly useful in situations where the data exhibits different behaviors or regimes at different points in time. Markov switching models assume that the data follows different statistical distributions or processes in different "states" or "regimes." These states are assumed to evolve over time following a Markov process, meaning that the probability of transitioning from one state to another depends only on the current state. In MS models discrete changes, in these patterns are integrated into a model to accommodate nonlinearities. These models, as described by Hamilton in 1990, differ from regression models with structural breaks because they assume that model parameters remain constant within a finite number of recurring patterns, even though these patterns may not be directly observable. Additionally, MS models are well-suited for modeling the cyclic aspects of time series data.

The fundamental concept in this model is that the changes in macroeconomic time series, which are measured using logarithms, do not conform to a linear stationary process as suggested by Hamilton in 1989. Instead, they exhibit a non-linear stationary pattern. Previous research on models of this nature can be credited to Goldfeld and Quandt in 1973, Maddala in 1986, Hamilton in various works from 1989 to 1996, and Fruhwirth-Schnatter in 2006. In our present research, we are focused on a specific variable known as the All-share index (ASI). This variable is influenced by a process that is reliant on the values of a hidden discrete state variable referred to as "r_t." We make an assumption that there exist G different states or conditions, and each of these states is characterized by a value of "*g*" that can range from 1 to G. A crucial premise of this model is that each state or condition possesses its own unique and exclusive regression pattern. This concept of distinct regression patterns for each state is a fundamental aspect of our analysis when examining shifts or changes in these states.

3.1. Mathematical aspect of Markov Switching (MS)

An MS is created by expanding upon a simple regime/state framework (*see* Alabi and Bada, 2022) which involves a basic switching model expressed as:

$$\mu_t(g) = X_t^{'}\theta_g + \Gamma_t^{'}\delta \tag{1}$$

In this model, we have two sets of coefficient vectors: θ_g, which is a $\boldsymbol{k}_x$ vector of coefficients, and δ, which is a $\boldsymbol{k}_z$ vector of coefficients. The δ coefficients relate to Γ_t and are consistent across different regimes, meaning they don't change with the regime. We also assume that the errors in the model follow a normal distribution, and the variances of these errors vary depending on the specific regime, denoted as "*g*." Also, an optimization of normal mixture log-likelihood function (NMLLF) expressed as:

$$l(\theta,\delta,\sigma,\gamma)=\sum_{t=1}^{T}\log\left\{\sum_{g=1}^{G}\frac{1}{\sigma_g}\phi\left(\frac{y_t-\mu_t(g)}{\sigma(g)}\right)p_g\left(U_{t-1,}\gamma\right)\right\} \tag{2}$$

was carried out. The basic switching model was estimated by obtaining the maximum likelihood estimators of the parameters in equation (2) and may be achieved through methods such as Marquardt steps and iterative techniques like Broyden, Fletcher, Goldfarb, and Shanno (BFGS). The parameters involved include; **θ**: A vector containing coefficients specific to each of the G regimes, **δ**: A vector of coefficients that remain consistent across all regimes, **σ**: A vector of coefficients representing the standard deviations of errors for each regime and **γ**: Parameters that play a role in determining the probabilities of transitioning between different regimes. U_{t-1} are exogenous variables that influence the regime probabilities. They are determined based on information available up to the previous time period. The inverse of the negative Hessian matrix is used to calculate the covariances of the model coefficients, providing insights into the parameter uncertainties. Equation (2) relies on the probabilities of being in a specific state one-step ahead, where "*g*" can take on values from 1 to G. It's important to note that filtering and smoothing methods can also influence these regime probabilities. Filtering is the process of updating regime probability estimates in real-time as new information becomes available about the values of the dependent variable in a specific time period. This means that as more data is observed, the probabilities of being in different regimes can change. Additionally, smoothing techniques can also impact these regime probabilities. Smoothing is a process of revising regime probability estimates retrospectively, using all available data up to a particular point in time. Introducing Bayes' theorem, a fundamental concept in probability theory and statistics, can help in these processes by incorporating new data and adjusting the regime probabilities accordingly to improve the model's predictions. We have

$$P(r_t=g\mid\varsigma_t)=P(r_t=g\mid y_y,\varsigma_{t-1})=\frac{f(y_t\mid r_t,\varsigma_{t-1})P(r_t=g\mid\varsigma_{t-1})}{f(y_t\mid\varsigma_{t-1})} \tag{3}$$

$$P(r_t=g\mid\varsigma_t)=\frac{\frac{1}{\sigma_g}\phi\left(\frac{y_t-\mu_t(g)}{\sigma(g)}\right)p_g(U_{t-1},\gamma)}{\sum_{j=1}^{G}\frac{1}{\sigma_j}\phi\left(\frac{y_t-\mu_t(j)}{\sigma(g)}\right)p_j(U_{t-1},\gamma)}$$

This postulated simple switching model includes three variables as regressors: exchange rate (*exr*), crude oil price (*cp*), and inflation rate (*ir*). These three variables are part of a matrix of regressors called $\boldsymbol{X}_t$. Markov switching is an extension of the basic switching model in which 1st-order Markov processes is included to determine regime probabilities. This means that the probabilities of being in a particular regime (let's call it "*g*") depend on the regime that was observed just before it. In simpler terms, whether we are in regime "*g*" or not is influenced by the regime we were in during the previous time period. This 1st-order Markov process introduces a level of dependence, as the current regime's probability is conditional on the previous regime, thus reflecting the idea that regime transitions have a certain degree of memory or persistence. In this approach, the probabilities of being in a particular regime (let's say "*g*") are determined by a 1st-order Markov process. This means that whether we are in regime "*g*" or not is influenced by the regime that was observed immediately before it. In other words, the likelihood of being in a particular regime depends on the regime that came just before it. This reflects the idea that regime transitions have a certain degree of dependence on the previous regime, and this dependence is a fundamental aspect of the 1st-order Markov process.

$$P\left(r_t = j \mid r_{t-1} = i\right) = P_{ij}(t)$$

$P_{ij}(t)$ are assumed to be equal notwithstanding time *t* i.e. $P_{ij}(t) = P_{ij}$ for all *t*. Here,

$$P^1(t) = \begin{bmatrix} p_{11}(t) & . & . & . & p_{1g}(t) \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ p_{g1}(t) & . & . & . & p_{gg}(t) \end{bmatrix}$$

where ij^{th} element represents the chance of transiting to state *j* in period *t* from state *i* in period *t*-1. $P\left(r_t = j \mid r_{t-1} = i\right) = P_{ij}(t)$ can be written as

$$P_{ij}(U_{t-1}, \gamma_i) = \frac{\exp(U_{t-1}'\gamma_{ij})}{\sum_{s=1}^{G} \exp(U_{t-1}'\gamma_{is})} \tag{4}$$

For all $j = 1, 2\ldots G$, $i = 1, 2\ldots G$, $\gamma_{iG} = 0$.

One property of MS of $p_{ij}(t)$ is that the full log-likelihood function is estimated recursively by filtering probabilities, $P\left(r_{t-1} = g \mid \varsigma_{t-1}\right)$.

3.2. Smoothing of one-step ahead regime probabilities

In contrast to filtered estimates, which rely on facts available at the same time as the estimation (ς_t), smoothed estimates for state probabilities at time *t* utilize data up to ς_t. According to Kim (2004) and Kim and Nelson (1999), efficient smoothing can be achieved through specific algorithms that involve backward computations through the data. These algorithms are designed to calculate joint probabilities efficiently, allowing for more accurate and comprehensive smoothing of regime probabilities.

$$P(r_t = i, r_{t+1} = j \mid \varsigma_T) = \frac{P(r_t = i, r_{t+1} = j \mid \varsigma_t)}{p(r_{t-1} = j \mid \varsigma_t)} P(r_{t+1} = j \mid \varsigma_T) \tag{5}$$

The smoothed probabilities at time *t* are obtained by marginalizing the joint probability w.r.t r_{t+1} such that

$$P(r_t = i \mid \varsigma_T) = \sum P(r_t = i, r_{t+1} = j \mid \varsigma_T) \tag{6}$$

3.3. Markov Switching Autoregressive MSAR(*p*)

The MS model can be adapted to comprise lagged integral factors and errors that exhibit serial correlation. By expanding the basic state context, which incorporates a 1st-order Markov process where transition probabilities depend on the prior state, we assume that the errors were serially uncorrelated. However, if the errors do exhibit serial correlation, Hamilton postulated an autoregressive (AR) specification with serial correlation of order "*p*" as follows.

$$y_t = \mu_t(r_t) + \sum_{m=1}^{p} \rho_m(r_t)(y_{t-m} - \mu_{t-m}(r_{t-m})) + \sigma(r_t)\varepsilon_t \tag{7}$$

The AR model essentially captures the serial dependence in the errors up to a lag of "*p*" as follows Equation (7) is commonly recognized as the MS autoregressive (MSAR) model. In this model, the mean equation is a central component.

$$\mu_t(g) = X_t^{'}\theta_g + \Gamma_t^{'}\delta + \sum_{m=1}^{k} \varphi_{mg} y_{t-m} \tag{8}$$

Based on the equation of the mean, a MSAR(*p*) is expressed as

$$(1 - \sum \rho_r(r_t) L^{\tau})(y_t - \mu_t(r_t)) = \sigma(r_t)\varepsilon_t \tag{9}$$

Due to the existence of state-dependent lagged mean, we require the probabilities for $p + 1$ dimensional regime vectors for the present and p preceding states in order to derive the NMLLF. The NMLLF for an AR (p) specification is given as

$$l(\theta, \delta, \sigma, \gamma, \rho) = \sum_{t=1}^{T} \log\left\{ \sum_{i=1}^{p} \sum_{j=1}^{p} \frac{1}{\sigma(i)} \phi\left(\frac{y_t - \mu_t(i) - \rho_g(i)(y_{t-1} - \mu_{t-1}(i))}{\sigma(i)} \right) p(r_t = i, r_{t-1} = j \mid \varsigma_{t-1}) \right\} \tag{10}$$

Equation (10) requires $p + p$ potential state outcomes probabilities for the regime vector $(r_{t,} r_{t-1})$ with $G^{*} = G^{p+1}$ potential realizations.

3.4. Filtering and Smoothing of MSAR(p)

In the literature, efficient approaches for estimating statistical parameters in lagged-state nonlinear filtering have been presented. The objective of these approaches is to maximize the normal mixture log likelihood function. Hamilton, in particular, presented a filter in 1989 that is aa modified MS filter. It supports a $p + 1$ dimensional state space and operates on similar concepts. However, the probability for delayed values of the states (r_{t-1}, r_{t-2}, r_{t-3},...) conditional on the data at time t-1 in this filter are affected by both the prior iteration of the filter and the one-step forward joint probabilities. In 1989, Hamilton also presented the Hamilton smoother, a modified lag-state smoothing method. This filtering process, according to Hamilton in 1990 resolved the problems associated with statistically finding "turning points" or cycles in time series data. Kim enhanced this process in 1994 by creating the Kim smoother, an effective smoothing and filtering procedure. It uses a single backward recursion step to handle a vector of possibilities denoted as G*. The addition of lag states to the MSAR specification modifies the estimating procedure. It entails assessing a vector of state variables representing the current and delayed states (p + 1 in total). The parameters in the MSAR specification are estimated by considering the model as a limited MS model with transition probabilities independent of the origin state. These transition probabilities are altered in such a way that the transition matrix's rows are indistinguishable.

$$P\left(r_t = j \mid r_{t-1} = i\right) = p_{ij}(t) = p_j(t) \tag{11}$$

and

$$P(t) = \begin{bmatrix} p_1(t) & . & . & . & p_G(t) \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ p_1(t) & . & . & . & p_G(t) \end{bmatrix}$$

To estimate one-step ahead, likelihood, filtered, and smoothed values for the restricted MS Model specification, we use the Kim Smoother. First, we compute the probabilities for the vector of probabilities associated with the $G^* = G^{p+1}$ dimensional state vector. Next, we set the initial probabilities in period ($-p$ + 1) using steady-state values, also known as the ergodic model.

4. RESULTS AND DISCUSSION

Starting with the All-Share Index, Exchange Rate (*exr*), Crude Oil Price (*cp*), and Inflation Rate (*ir*) time plots (Figure 1), secondary data for five chosen macroeconomic indicators are shown before the analysis is conducted. This information, which spanned the years from 2006 to 2022, was taken from a database maintained by the Central Bank of Nigeria (CBN). Two datasets were created from the data: 96% of data, covering the years 2006 to 2021, was utilized to train the time-varying MS model, and the final 4%, covering the months of January to September 2022, was set aside for forecasting. Log differences were generated to make the time series data appropriate for analysis. The time series data were transformed since it was thought they displayed nonstationary behavior. The state or regime is referred to as the "market" for convenience's sake. The dependent variable in this time-varying model, the All-share Index, is split into "bullish" and "bearish" markets. The "bearish" market suggests a decline in share prices as opposed to the "bullish" market, which implies an increase in share prices. In this time-varying model, regime 1 represents the bearish market, and regime 2 the bullish market. The evolution of these markets was represented using a first-order Markov process. This modeling method, which is based on the Markov switching framework, was utilized to analyze and comprehend the Nigerian stock market's behavior, particularly with regard to the All-share Index, and to predict transitions between bullish and bearish markets. With the help of the Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimization method, the model was estimated with Marquardt steps. This optimization method was used with two states (G = 2; regime 1 = bearish market and regime 2 = bullish market). The observed inverse negative Hessian scheme was used in the computation of the covariance and standard errors of the model parameters, which is a common technique for estimating parameter uncertainties. Different initial values for the model parameters were employed during the optimization process, which entailed a random search. The L'Ecuyer random number generator was used to facilitate random search, and appropriate seeds were established to assure repeatability and uniformity across iterations. MS nonlinear iterative filtering served as the main optimization technique. In this method, the ergodic solution was solved for, allowing the calculation of the filtered initial one-step forward probability. The model's likelihood, a crucial step in the estimating process, was determined in large part by these probabilities.

Figure 1 presents a time plot of four variables: log(*asi*) (All-Share Index), log(*cp*) (Crude Oil Price), log(*exr*) (Exchange Rate), and log(*ir*) (Inflation Rate). These data were collected on a monthly basis from 2006 to 2022. The log ASI plot shows a cyclical pattern with a slightly positive trend and relatively low variability. It is noteworthy that the share prices reached their highest point in February 2008. During the period from 2006 to 2008, the ASI experienced significant growth. This growth was attributable to

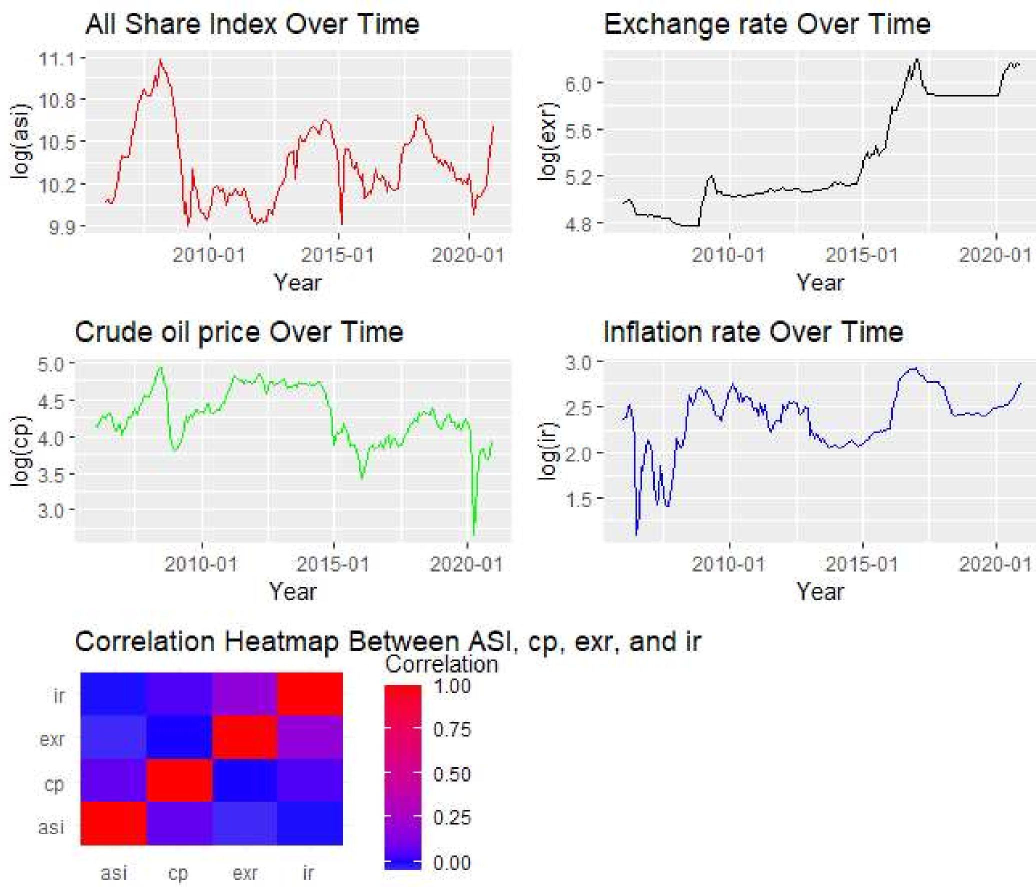

Figure 1: Time plots of log(*asi*), log(*cp*) and log(*exr*) and Correlation Heatmap

several factors, including the rise in oil prices, increased market discipline, greater consumer sophistication, and the influx of oil revenue into the banking system. However, in 2009, there was a sharp drop in the All-Share Index, and it remained relatively stable from April 2009 to November 2012. This stability was a response to the global financial crisis, the banking crisis in Nigeria (resulting in a substantial market cap reduction of 30 billion naira in a single day), and unfavorable interest rates (Adeolu, 2012). The plot of crude oil prices indicates a cyclical variation in the data, with the highest and lowest prices occurring in June 2008 and March 2020, respectively. Both the inflation rate and exchange rate plots exhibit fluctuating patterns with periodic ups and downs. These series show cyclical variations with an overall increasing trend and relatively low variability. The heatmap plot shows the correlations between these variables. Particularly, a positive correlation was observed between ASI and the three variables. Typically, a gradient color scale is used, with lighter and darker colors for low and high correlation values respectively. The color legend indicates the scale or degree of association between these variables. The weakest positive correlation value was observed between ASI and the inflation rate. This is shown by the light blue color intensity represents the value of each data point. In summary, Figure 1 provides a visual representation of the historical correlations, trends, and patterns in these four key macroeconomic variables over the analyzed time period, shedding light on their fluctuations and important events that influenced their trajectories.

4.2. Time-varying MSAR on All-share index

Under the presumption that there is no autocorrelation in the data, the time-varying Markov Switching Autoregressive (MSAR) model was estimated. The exchange rate at lag 1 was incorporated as a time-varying regressor. This time-varying independent variable is modeled to change in response to adjustments in the system's states or regimes and is thought of as endogenous. We maximized the NMLLF to calculate the model's parameters, including the coefficients for the time-varying regressors. The computed coefficients show how this time-varying regressor's effects change based on the system's state. One standard deviation was used throughout the optimization procedure, which involved 100 initial values and 100 iterations. A random number generator called the L'Ecuyer random number generator was used to ease this procedure, and a precise seed value of 346,892,529 was chosen to guarantee consistency and reproducibility in the results.

Transition Probabilities and Expected Durations

$$P_{i,j} = P\,(r_t = j \mid r_{t-1} = i)$$

Average Transition Probabilities

$$p(t) = \begin{bmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.0000126 & 0.999987 \\ 0.057568 & 0.942432 \end{bmatrix}$$

Average Expected Duration

Bearish	Bullish
1.000013	18.4244

The transition probability matrix and predicted durations revealed a strong dependency on the state in the transition probabilities, with high possibilities of remaining in the origin for bearish (0.0000126) and bullish (0.942432). Also, the probabilities of switching from bearish to bullish and bullish to bearish were (0.999987) and (0.05757). The overall market movement is expected to spend about 18 months and 4 weeks in the 'bullish' state and a month when in the bearish state.

Table 1: Time-varying MSAR coefficient, standard error, z-statistics and *p*-value for bearish

Variable	*Coefficient*	*S.E*	*z-Statistic*	*p-value*
$\Delta\log(asi_{t-1})$	1.067	0.110	9.672	0.000*
$\Delta\log(exr_t)$	194.500	45.392	4.285	0.000*
$\Delta\log(exr_{t-1})$	207.984	68.430	3.039	0.002*
$\Delta\log(exr_{t-2})$	-336.930	210.060	-1.604	0.109
$\Delta\log(exr_{t-3})$	-69.063	178.229	-0.387	0.698
$\Delta\log(ir_t)$	-488.809	388.419	-1.258	0.208
$\Delta\log(cp_t)$	557.843	87.730	6.359	0.000*
$\Delta\log(cp_{t-1})$	-461.187	87.473	-5.272	0.000*

Note: * indicates co-efficient significant at 1% level of significance.

Table 1 shows regression statistics for a Markov Switching Autoregressive (MSAR) model in the context of bearish regime. Each row in the table corresponds to a variable, and the columns provide information about the coefficient estimates, standard errors, *z*-statistics, and *p*-values for that variable within the bearish market. In this bearish market, the estimated coefficient for the variable $\Delta\log(\mathbf{asi}_{t-1})$ is 1.067. This indicates that, within this bearish market, a one-unit increase in the $\Delta\log(\mathbf{asi}_{t-1})$ is associated with an average increase of approximately 1.067 units in the current $\Delta\log(\mathbf{asi}_t)$. The *p*-value associated with this coefficient is very close to zero (0.000), indicating strong evidence

Table 2: Time-varying MSAR coefficient, standard error, z-statistics and p-value for bullish market

Variable	*Coefficient*	*S.E*	*z-Statistic*	*p-value*
$\Delta\log(asi_{t-1})$	0.951	0.023	41.311	0.000*
$\Delta\log(exr_t)$	-15.952	12.837	-1.243	0.214
$\Delta\log(exr_{t-1})$	15.569	26.278	0.592	0.554
$\Delta\log(exr_{t-2})$	-58.343	54.759	-1.065	0.287
$\Delta\log(exr_{t-3})$	65.515	39.772	1.647	0.100
$\Delta\log(ir_t)$	-52.149	77.705	-0.671	0.502
$\Delta\log(cp_t)$	102.707	37.055	2.772	0.006*
$\Delta\log(cp_{t-1})$	0.951	0.023	41.311	0.000*

Note: *indicates co-efficient significant at 0.01 level of significance.

against the null hypothesis. This further supports the notion that the "Δlog(**asi**$_{t-1}$)" variable is highly significant within this regime. The table provide similar information for other variables, each of which represents the effect of a specific lagged or contemporaneous variable affect the All-Share Index within the bearish market of the Markov Switching Autoregressive (MSAR) model. The statistics help determine the statistical significance and magnitude of these effects in this specific market state.

Table 2 shows the time-varying MSAR assuming no serial autocorrelation in the bullish state of the stock market. It indicated that there was a negative relationship between exchange rate, exchange rate lag 2, inflation rate, and All-share index but these relationships were not statistically significant at all levels of significance. This implies that the current exchange rate, lag 2 exchange rate, and the inflation rate would reduce the stock market value on average by 15.95, 58.34, and 52.15 respectively. However, a positive relationship was established between All-share index at lag 1, exchange rate at lag1 and lag 3, crude oil price, crude oil price at lag1 and All-share index, which was significant for All-share index at lag 1, crude oil price and crude oil price at lag one at all levels, indicating that immediate past value of All-share index, immediate past exchange rate, 3 days past period exchange rate, and current value and immediate past values of crude oil price would positively affect the stock market on the average by 0.951, 15.57, 65.52, 102.71 and 0.951 respectively.

Table 3: Transition matrix parameters

Variable	*Coefficient*	*S.E*	*z-Statistic*	*p-value*
P_{11}-C	-0.447	1.132	-0.395	0.693
P_{11}-Δlog(exr$_{t-1}$)	-0.074	0.010	-7.283	0.000*
P_{21}-C	-2.284	1.417	-1.612	0.107
P_{21}-Δlog(exr$_{t-1}$)	-0.002	0.006	-0.374	0.708

Note: *, ** indicate co-efficient significant at 0.01 and 0.05 levels of significance respectively.

Table 3 contains transition matrix parameters for a Markov switching model used for predicting the All-Share Index in Nigeria. In this model, P_{11} and P_{21} represent the transition probabilities between bearish and bullish markets, and the coefficients associated with C and Δlog(**exr**$_{t-1}$) are used to model the transition probabilities. **Δ**log(**exr**$_{t-1}$) is a time-varying regressor related to the exchange rate. The coefficients in the transition matrix parameters indicate how the presence of constant terms and the time-varying regressor Δlog(**exr**$_{t-1}$) influence the transition probabilities between different states in the Markov switching model. The significant negative coefficient for P_{11}-Δlog(**exr**$_{t-1}$) suggests that changes in the exchange rate at lag 1 variable is associated with changes in the likelihood of staying in bearish market. However, the low *p*-value indicates that this effect is statistically significant. The other coefficients have less statistically significant effects. This unfavorable impact corresponds with the discoveries made by Yucel and Kurt in 2003, who proposed that the stock market in export-focused nations is negatively influenced by fluctuating exchange rates. Various elements, including shifts in currency rates, can influence variables in both economies and enterprises. The consequences of these fluctuations in exchange rates on existing businesses differ based on the economic conditions in different countries. For instance, companies that import raw materials but primarily sell their finished products in their local market are more likely to encounter financial challenges when the currency's value declines. Conversely, if these companies sell a substantial portion of their finished products internationally, a drop in the currency's value can actually be financially advantageous for them.

The fluctuations in exchange rates hold equal importance for businesses not engaged in international trade. The influence of the inflation rate on the stock market's performance revealed both positive and negative tendencies, although neither of these relationships was statistically significant. The positive correlation supports Fisher's hypothesis, which suggests that equity stocks, representing claims against a company's assets, may serve as a safeguard when inflation unexpectedly surges. This implies a positive connection between stock prices and inflation. In the model's bullish and bearish markets, the price of crude oil exhibited both positive and negative effects. These findings are in line with the research conducted by Cong, Weiy, Jioa, and Fan in 2008, Sadorsky in 2001, and El-sharif, Brown, Burton, Nixon, and Russell in 2005, all of which support a positive association between crude oil prices and the stock market. Conversely, the inverse relationship supports the notion that an increase in crude oil prices might have an adverse impact on the stock market. This is primarily due to investors' reactions to the inflationary nature of rising oil prices, which could affect their overall earnings. Consequently, the stock market is susceptible to the detrimental consequences of higher oil prices on corporate profitability, suggesting that the market may eventually become bearish.

Table 4: Common parameter

Variable	*Coefficient*	*S.E*	*z-Statistic*	*p-value*
log (σ)	7.761	0.124	62.470	0.000*

Note: * indicates co-efficient significant at 0.01 level of significance.

Similarly, Table 4 displayed the result of the common parameter, used as non-switching regressors in the MSAR model. The common parameter is crucial as it

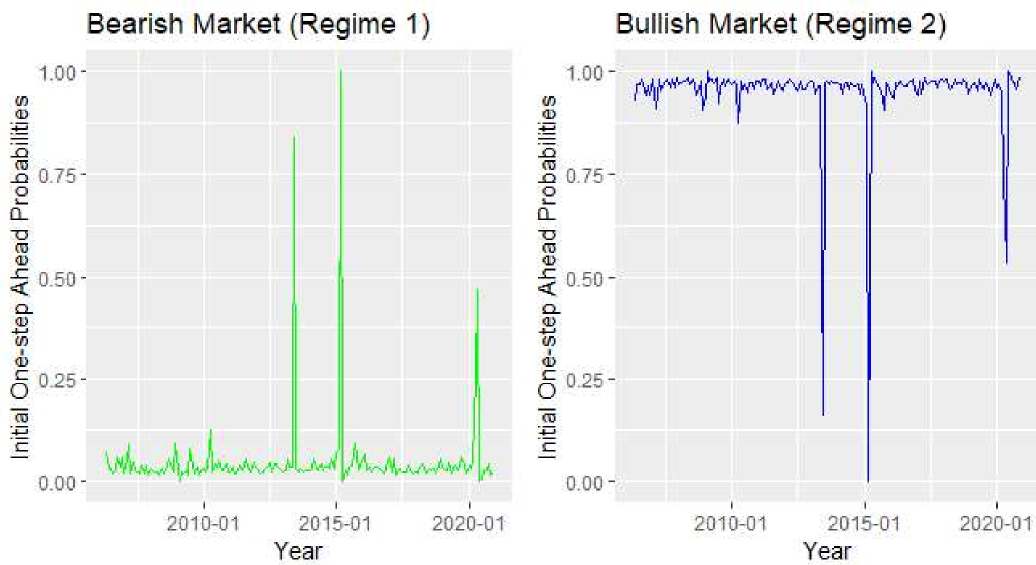

Figure 2: Markov-switching one step ahead of initial transition probabilities

represents the aspect of the model that is consistent across different bearish and bullish markets. The statistical significance of this parameter helps assess its importance in explaining the behavior of the system regardless of its current state. A very low *p*-value suggests strong evidence against the null hypothesis, indicating that the common parameter is statistically significant regardless if the market is bearish or bullish.

Figure 2 displays plots depicting the initial one-step-ahead probabilities. These plots illustrate the likelihood that the All-Share Index, representing the Nigerian stock market, will be in either a bearish or bullish market at time period "*t*." This assessment is based on the information available at ς_{t-1}. The plots reveal distinct patterns in the time-varying model, when the probabilities of the state being in a "bear" market increase at a specific time period "*t*," the probabilities of it being in a "bull" market decrease. Furthermore, there appears to be an inverse relationship between the two markets in the time-varying model. These plots and probabilities provide insights into how the market, either "bullish" or "bearish," evolves over time and how changes in these markets are influenced by the available information at each time period.

Figure 3 presents plots illustrating the filtered and smoothed probabilities of the initial one-step ahead probabilities in Figure 2. These plots showcase the likelihood of the ASI, being in either an increasing or declining state at period "*t*" after filtering and smoothing of the filtered probabilities. These assessments are made based on the data available at the preceding time period ς_{t-1}. The key observation from these plots is similar to the observations on the initial one-step ahead probability plots. These patterns

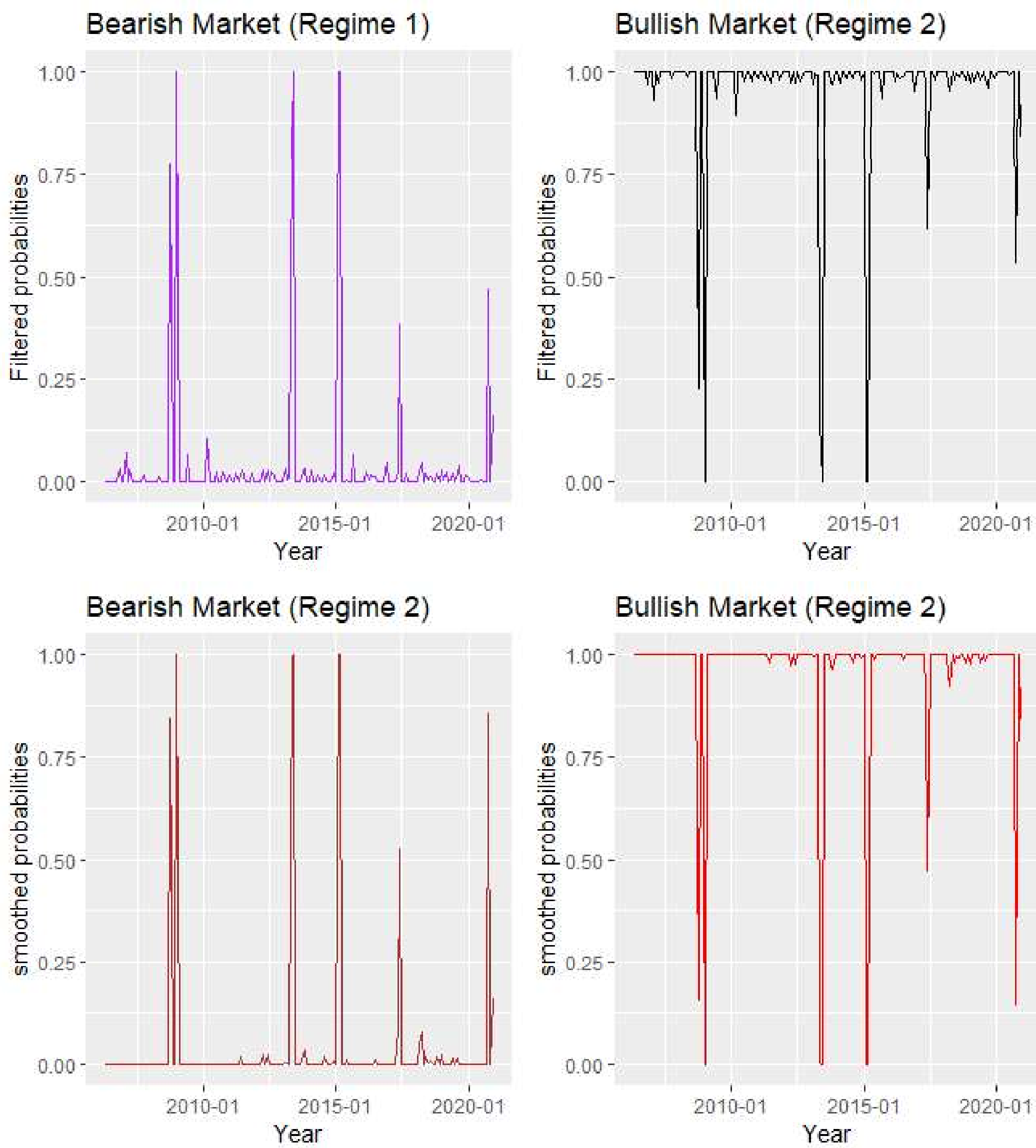

Figure 3: Markov-switching filtered and smoothed probabilities

suggest that the inverse relationship between the two markets at a particular period does not change after filtering and smoothing. These plots and probabilities offer valuable insights into how the stock market shifts between "bullish" and "bearish" markets over time and how these drifts are influenced by the available market information at each time period. Summarily, using the data available at the time period t-1, these graphs illustrate the chance that the ASI, is rising or dropping at time period t. These

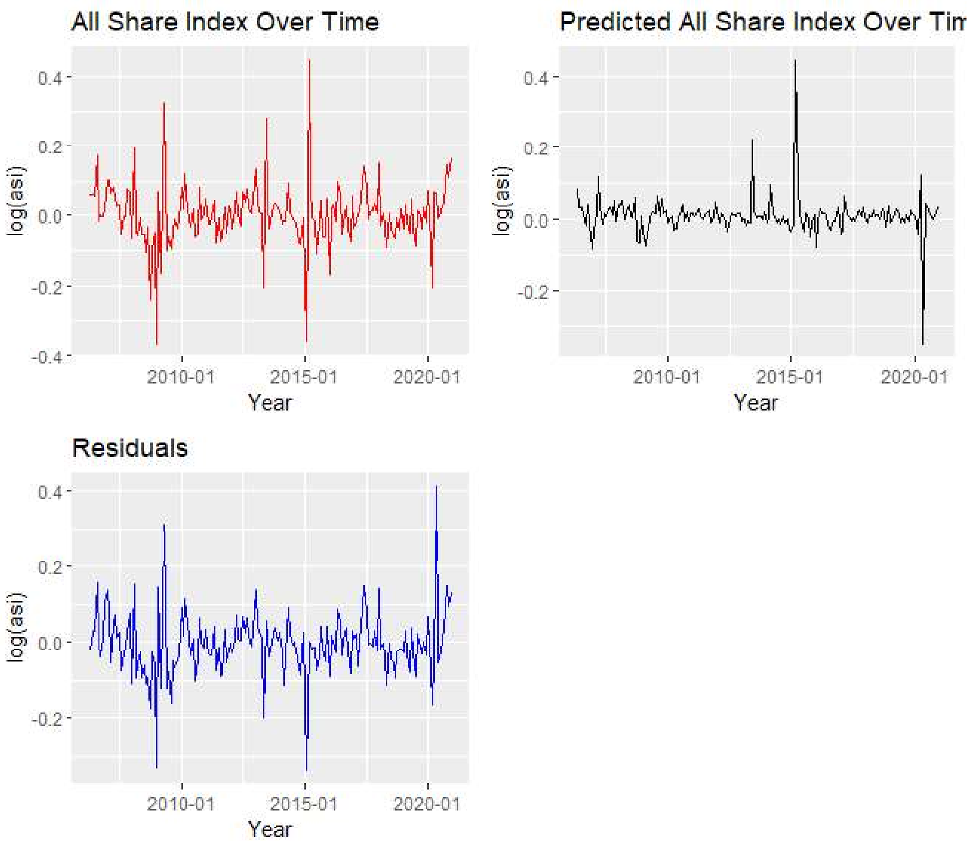

Figure 4: Residuals. Predicted and actual values of log (ASI) (2006-2021)

charts show that as the probability of the bearish market increases, the probability of the bullish market decreases at a certain time period *t*.

Figure 4 depicts the residual values of the All-share index from 2006-2021. It showed that the deviations between the actual and predicted were relatively small, indicating the model gave a nearly accurate prediction. These can be observed in the values of the variance, covariance proportion of 15.6% and 83.9% respectively. Furthermore, the bias proportion value of 0.57% indicates that the predicted and actual values are moving closely together. Our model is considered adequate for forecasting purposes since it incorporates all relevant information and calibration to actual data indicating no important significant departures from the statistical assumptions.

We conducted a forecast for the All-Share Index spanning from January to September 2022, as illustrated in Figure 5. This graph demonstrates that the MSAR model exhibited its strongest predictive performance from July to September 2022 but had less accurate predictions during the initial half of the year.

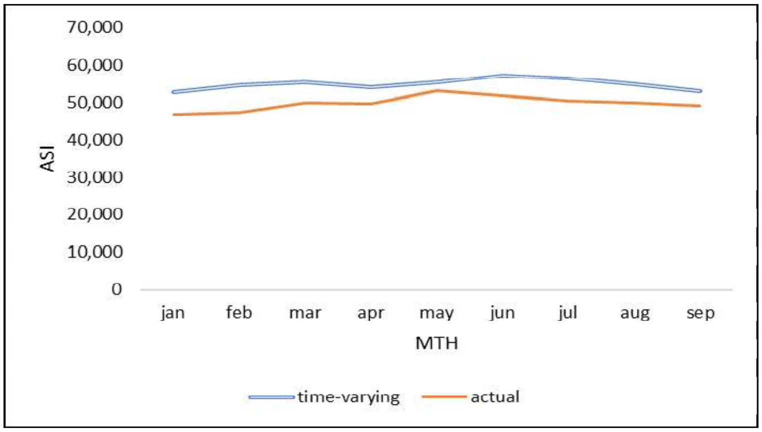

Figure 5: Forecasted values of ASI for the period (January – September 2022)

5. CONCLUSION

In line with the study's objectives, the statistical significance observed in the selected macroeconomic variables can be attributed to the exchange rate (which had a bearish impact) and crude oil prices (with both bearish and bullish effects). These variables exhibited a positive relationship with the all-share index. The transition probability matrix revealed noteworthy dependencies on the markets for the transition probabilities. Specifically, there were both low and high probabilities of remaining in the bearish market and the bullish market, respectively. Furthermore, the probability of switching from a bearish to a bullish market and vice versa was found to be mixed. The smoothed market probabilities depicted turning points or shifts in the market sentiment during various periods, notably in January 2008, November-December 2008, April-May 2009, April 2013, March 2015, and January-March 2021.

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